Measuring Im(λ_t) using $K_S - K_L$ Interference

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1. Introduction

This note is inspired by the discussion in section 3 of ref [1]. This reference discusses measuring Im(λ_t) by observing $K_S - K_L$ interference in the K^0 time evolution. This note is an attempt to expand on this section a bit. I attempt here to derive the relevant equations. Using toy simulations and a fitter, I estimate the number of $\pi^0 e^+ e^-$ events needed to extract Im(λ_t) to 10%. I also point out a curious case for which one cannot extract Im(λ_t). Let:

$$\begin{aligned} a_S &= <\pi^0 ee|H|K_S>= |a_S|e^{i\phi_S} \\ a_L &= <\pi^0 ee|H|K_L>= |a_L|e^{i\phi_L} \\ |a_S|^2 &= \Gamma(K_S)BR(K_S \to \pi^0 ee) \\ |a_L|^2 &= \Gamma(K_L)BR(K_L \to \pi^0 ee) \end{aligned}$$

From ref [2]:

$$BR(K_L \to \pi^0 ee)\cdot 10^{12} = 15.3(A_S^2) - 6.8A_S\mathrm{Im}(\lambda_t) + 2.8\mathrm{Im}^2(\lambda_t) \tag{1}$$

where Im(λ_t) is given in units of 10^{-4}, and the CPC term has been safely ignored.

From the NA48 $K_S \to \pi^0 ee$ discovery [3]:

$$\begin{aligned} BR(K_S \to \pi^0 ee) &= (A_S^2)5.2\cdot 10^{-9} \\ A_S &= \pm 1.06^{+0.26}_{-0.21} \pm 0.07 \end{aligned} \tag{2}$$

For a pure K^0 ($\overline{K^0}$) initial state, the time evolution is:

$$|K^0(t)> \sim |K_S> e^{-im_S t - \frac{t}{2t_S}} + |K_L> e^{-im_L t - \frac{t}{2t_L}}$$

$$|\overline{K^0(t)}> \sim |K_S> e^{-im_S t - \frac{t}{2t_S}} - |K_L> e^{-im_L t - \frac{t}{2t_L}}$$

where $t_S = K_S$ lifetime and $t_L = K_L$ lifetime.

The number of $\pi^0 ee$ events per unit propertime (t) versus propertime is:

$$\frac{dN}{dt} \sim |a_S e^{-im_S t - t/(2t_S)} + a_L e^{-im_L t - t/(2t_L)}|^2$$

Similarly, for a pure $\overline{K0}$ initial state:

$$\frac{dN}{dt} \sim |a_S e^{-im_S t - t/(2t_S)} - a_L e^{-im_L t - t/(2t_L)}|^2$$

Note that the $K_L - K_S$ interference for pure K^0 and $\overline{K^0}$ initial state differs by a minus sign. So for the case of neutral kaons produced by protons on target, where the initial kaon state is not generally tagged, the interference amplitude will be reduced by the dilution factor D.

$$D = \frac{N(K^0) - N(\overline{K^0})}{N(K^0) + N(\overline{K^0})} = 0.3$$

D is the dilution factor, which depends on certain parameters of the secondary beam. For demonstration, I will use a (reasonable) value of D = 0.3. Then doing some algebra, including the dilution:

$$\frac{dN}{dt} = |a_S|^2(e^{-t/t_S} + 2D\frac{|a_L|}{|a_S|}cos((m_L - m_S)t + \phi_S - \phi_L)e^{\frac{-t(t_S+t_L)}{2t_S t_L}} + \frac{|a_L|^2}{|a_S|^2}e^{-t/t_L}) \tag{3}$$

So the idea is that by observing the time-evolution of dN/dt, one can extract $|a_L|/|a_S|$, which can ultimately be related to Im(λ_t) by equations 1 and 2.

We must also include the $K_L \to ee\gamma\gamma$ background, which adds incoherently to equation 3. The $K_S \to ee\gamma\gamma$ is negligible. I shall assume an effective BR($K_L \to ee\gamma\gamma$) = 10^{-10} after cuts. This is about the value achieved by the KTeV $K_L \to \pi^0 ee$ search.

Let $|a_L|^2 = \Gamma(K_L)BR(K_L \to ee\gamma\gamma)$ and

$$\frac{dN}{dt}(K_L \to ee\gamma\gamma) = |a_L(ee\gamma\gamma)|^2 e^{-t/t_L} \tag{4}$$

Combining equations 3 and 4 and simplifying some terms:

$$\frac{dN}{dt} \sim e^{-t/t_S} + 2D\sqrt{R}\cdot\cos(\Delta\mathrm{m}+\Delta\phi)\mathrm{e}^{-\mathrm{t}(\mathrm{t_S}+\mathrm{t_L})/(2\mathrm{t_S}\mathrm{t_L})} + (R+R_{ee\gamma\gamma})e^{-t/t_L} \quad (5)$$

where D=0.3 (dilution factor), $\Delta m = m_L - m_S$, $\Delta\phi = \phi_S - \phi_L = 57°$ [4], and

$$R = [15.3 - 6.8\frac{\mathrm{Im}(\lambda_\mathrm{t})}{A_S} + 2.8(\frac{\mathrm{Im}(\lambda_\mathrm{t})}{A_S})^2] \quad (6)$$
$$\cdot 10^{-12}\frac{t_S}{t_L}\frac{1}{5.2\cdot 10^{-9}}$$
$$R_{ee\gamma\gamma} = 10^{-10}\frac{t_S}{t_L}\frac{1}{A_S^2 5.2\cdot 10^{-9}}$$

A plot of equation 5 is shown in figure 1 for propertimes between 6 and 16 t_S. At earlier t_S, the resolution on $\mathrm{Im}(\lambda_t)$ is worse. Figure 2 shows the rate for various values of $\mathrm{Im}(\lambda_t)$.

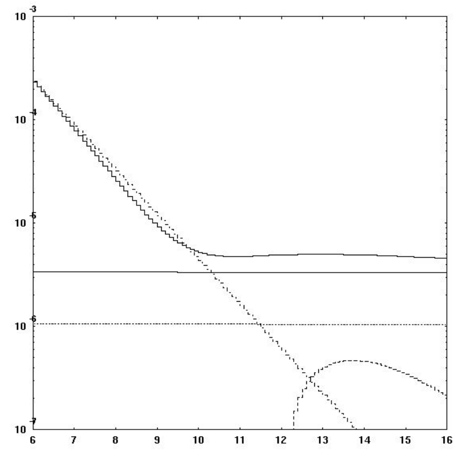

Figure 1. Number of events versus propertime t in units of t_S. The pure fast exponential is the K_S decay component. The two nearly horizontal lines are the pure K_L decays into $\pi^0 ee$ and $ee\gamma\gamma$. The bottom most curve is the interference term. The top curve is the combined.

2. Fit Result

I simulate an experiment by randomly sampling the parent distribution of equation 5, and fit to the result. The fit is done for 2 parameters: the amplitude of the cosine term and the overall normalization. This is the best case scenario, in which the dilution and $ee\gamma\gamma$ have been determined through other methods and therefore are fixed in the fit. Figure 3 shows a typical fit and error on $\mathrm{Im}(\lambda_t)$. The minimization package is the CERN Minuit package interfaced to PAW.

Two interesting features are noteworthy. First, my fit systematically biases $\mathrm{Im}(\lambda_t)$. See table 1 for the fit results. By disabling the statistical fluctuations or by increasing the sample size, I recover the proper input value of $\mathrm{Im}(\lambda_t)$. So I attribute this bias to finite statistics.

The second feature is the extraction of $\mathrm{Im}(\lambda_t)$ from R, which is a solution to the quadratic equation of 6. R is the parameter fitted from equation 5. Again, $\mathrm{Im}(\lambda_t)$ is understood to be in units of 10^{-4}.

$$\frac{R}{f} = [15.3 - 6.8\frac{\mathrm{Im}(\lambda_\mathrm{t})}{A_S} + 2.8(\frac{\mathrm{Im}(\lambda_\mathrm{t})}{A_S})^2]$$
$$f = 10^{-12}\frac{t_S}{t_L}\frac{1}{5.2\cdot 10^{-9}} \quad (7)$$

For certain values of $\mathrm{Im}(\lambda_t)/A_S$, the interference term and the direct CPV term cancel exactly. This seems to be purely coincidental in that the coefficients to the quadratic happens to conspire that way.

The uncertainty of $\mathrm{Im}(\lambda_t)$ is:

$$\frac{dR}{f} = [\frac{-6.8}{A_S} + 5.6\frac{\mathrm{Im}(\lambda_\mathrm{t})}{A_S^2}]d\mathrm{Im}(\lambda_t)$$

So for $\mathrm{Im}(\lambda_t)/A_S = 1.2$, there is almost no resolution on $\mathrm{Im}(\lambda_t)$. The current SM-preferred value is $\mathrm{Im}(\lambda_t)$=1.3. So for purely accidental reasons, if $A_S = 1.06$, we will have a very poor resolution on $\mathrm{Im}(\lambda_t)$.

A similar problem is seen in solving equation 7. Solving for $\mathrm{Im}(\lambda_t)$:

$$\frac{Im(\lambda_t)}{A_S} = \frac{1}{5.6}\cdot -6.8 \pm \sqrt{6.8^2 - 4\cdot 2.8\cdot(15.3-\frac{R}{f})}$$

For $\mathrm{Im}(\lambda_\mathrm{t})/\mathrm{A_S} = 6.8/5.6$, the term in the square

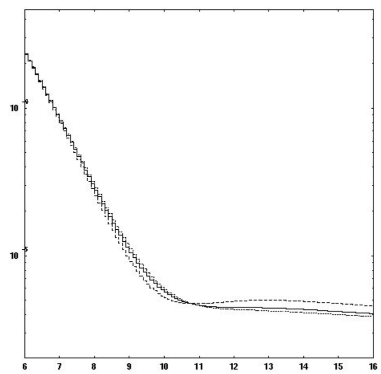

Figure 2. dN/dt versus t in units of t_S for $\mathrm{Im}(\lambda_t)$ $= \pm 1.3 \cdot 10^{-4}$ and 0.

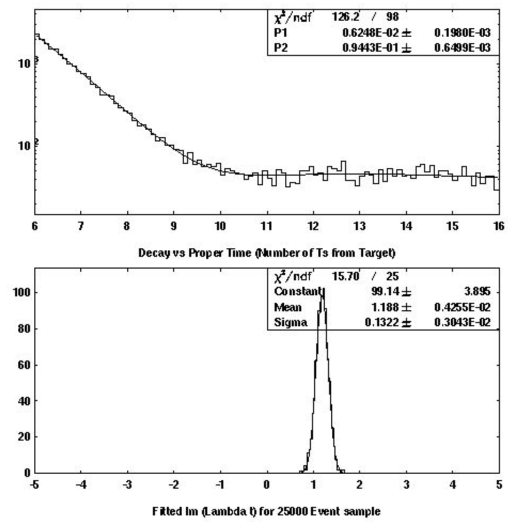

Figure 3. Upper plot shows a typical fit to an experiment with 25000 events. Lower plots shows the extracted $\mathrm{Im}(\lambda_t)$ for 1000 such experiments. Both plots are for the dilution D = 1, and the input $\mathrm{Im}(\lambda_t) = 1.3 \cdot 10^{-4}$, $A_S = -1.06$.

root vanishes exactly. So statistical fluctuations in R can return a non-physical $\mathrm{Im}(\lambda_t)/\mathrm{A_S}$. Again, this seems purely coincidental.

In this case of $\mathrm{Im}(\lambda_t)/A_S = 1.2$, we will have to appeal to a Bayesian-like statistical analysis to determine the most likely value for $\mathrm{Im}(\lambda_t)/A_S$. For the rest of the document, I will use $A_S = -1.06$, Im(λ)=1.3, thereby avoiding this region.

Table 1 lists the fit results, where I've used $A_S = -1.06$, Im(λ)=1.3, D=0.3.

Table 1
Fit results for $A_S = -1.06$, Im(λ)=1.3, and for neutral kaons from a target with dilution D=0.3. N is the sample size between 6 and 16 t_S.

N	Fitted $\mathrm{Im}(\lambda_t)$	Resolution on $\mathrm{Im}(\lambda_t)$
$25 \cdot 10^3$	0.97	0.22
$100 \cdot 10^3$	1.22	0.10
$400 \cdot 10^3$	1.28	0.05
10^6	1.29	0.03

3. Flux Estimate for the Case of Target Neutral Kaons

I can now estimate roughly the required number of "protons on target" (p.o.t). To good approximation, we need to observe about 10^5 decays to $\pi^0 ee$ and using only events at least 6 t_s from the target. Assume a detection efficiency of 5% and $\mathrm{BR}(K_S \to \pi^0 ee) = 5.8 \cdot 10^{-9}$.

Assume a proton energy of 120 GeV and beam targetting angle of 24 mrad. From Rick Coleman's program [5], 10^{12} (p.o.t.) yields $8.1 \cdot 10^6$ K_S produced at the target. Consider the extreme case where the fiducial volume starts at the target, and the decay volume length is sufficiently long that the geometrical acceptance is 100% for all K_S. In this extreme case, all decays greater than 6 t_S are used. From Rick Coleman's program, 10^{12} (p.o.t.) yields $1.9 \cdot 10^4$ usable K_S.

$$\mathrm{N(p.o.t.)} = \frac{10^{12} \cdot 10^5}{\mathrm{BR(K_S \to \pi^0 ee)} \cdot (0.05) \cdot 1.9 \cdot 10^4} = 1.8 \cdot 10^{22}$$

In practice, the detector volume must start at some minimum distance from the target. I shall take a wild guess of 10 meters ! If we wish to consider only kaons that have lived 6 t_S or less when they arrive at the detector volume, then these kaons have a minimum momentum P_{min} of:

$$P_{min} = (10\mathrm{m} \cdot \mathrm{M_K ct_S})/6 = 31\ GeV$$

With this requirement, 10^{12} p.o.t. yields $1.4 \cdot 10^3$ usable K_S. The required N(p.o.t.) becomes $2.4 \cdot 10^{23}$. This requirement could be lessened somewhat, as kaons below P_{min} still contain some usable information.

4. Coherently Regenerated K_S

In this section, I explore the case of coherently regenerated K_S, for example, generated by a K_L beam on a target. The initial state is $\rho K_S + K_L$, where $\rho = |\rho| \exp(i\phi_\rho)$ is the regeneration amplitude. Equations 5 and 6 becomes:

$$\frac{dN}{dt} \sim exp(-t/t_S) + 2\sqrt{R} cos(\Delta m + \Delta\phi) exp(\frac{-t(t_S + t_L)}{2 t_S t_L}) + (R + R_{ee\gamma\gamma}) exp(-t/t_L)$$

where $\Delta m = m_L - m_S$, $\Delta\phi = \phi_\rho + \phi_S - \phi_L = 45^\circ + 57^\circ$. And:

$$R = [15.3 - 6.8\frac{\mathrm{Im}(\lambda_t)}{A_S} + 2.8(\frac{\mathrm{Im}(\lambda_t)}{A_S})^2] \cdot 10^{-12}\frac{t_S}{t_L}\frac{1}{5.2 \cdot 10^{-9} \cdot |\rho|^2} \tag{8}$$

$$R_{ee\gamma\gamma} = 10^{-10}\frac{t_S}{t_L}\frac{1}{A_S^2 5.2 \cdot 10^{-9} \cdot |\rho|^2}$$

To get the best resolution on R, the optimal value for $|\rho|$ is when R = 1. For $A_S = -1.06$ and $\mathrm{Im}(\lambda_t) = 1.3$, the optimal is $|\rho|$=0.032. This is reasonably near existing devices such as the KTeV regenerator, for which $|\rho|$=0.04. Figure 4 shows dN/dt for case of regenerated K_S. In this case, the statistical power lies in events with small propertimes. The fit results for coherently produced K_S is shown in table 2. So for $N = 400 \cdot 10^3$, we get a 10% resolution on $\mathrm{Im}(\lambda_t)$.

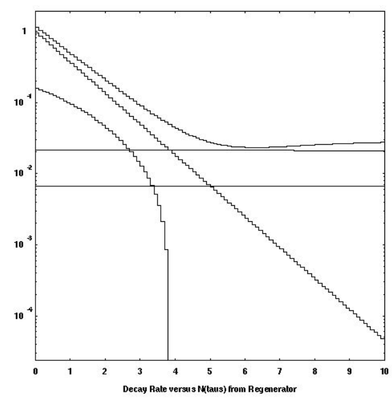

Figure 4. dN/dt versus t in units of t_S for coherently regenerated K_S. Simulation uses $|\rho| = 0.04$ and $\phi_\rho = 45^\circ$. The pure fast exponential is the K_S decay component. The two nearly horizontal lines are the pure K_L decays to $ee\gamma\gamma$ and $\pi^0 ee$. The bottom most curve is the interference term. The top curve is the combined.

Table 2
Fit results for $A_S = -1.06$, Im(λ)=1.3, and for $|\rho| = 0.032$ and $\phi_\rho = 45^\circ$. N is the sample size between 0 and 10 t_S.

N	Fitted $\mathrm{Im}(\lambda_t)$	Resolution on $\mathrm{Im}(\lambda_t)$
$25 \cdot 10^3$	0.94	0.34
$100 \cdot 10^3$	1.21	0.17
$400 \cdot 10^3$	1.28	0.08
10^6	1.29	0.06

5. Flux Estimate for the Case of Regenerated K_S

I can now estimate the required number of protons on target for the case of regenerated K_S. To good approximation, we need to observe $N = 400 \cdot 10^3$ $\pi^0 ee$ decays to have a 10% resolution on $\mathrm{Im}(\lambda_t)$. Assume a 5% efficiency, and the detector volume starts 90 meters from the primary target, from which K_L are produced. Assume that the regenerator target is at the start of the detector volume.

From the NA48 result:

$$BR(K_S \to \pi^0 ee) = 5.8 \cdot 10^{-9}$$

To produce this many K_S coherently, the number of K_L incident on the regenerator is:

$$N(K_L) = \frac{4 \cdot 10^5}{BR(K_S \to \pi^0 ee) \cdot 0.05 \cdot |\rho|^2} = 1.3 \cdot 10^{18} \qquad (9)$$

Assume proton energy of 120 GeV and beam targetting of 24 mrad. From Rick Coleman's program, 10^{12} p.o.t. yields $5.1 \cdot 10^6 K_L$ at 90 meters from the primary target.

$$N(p.o.t.) = \frac{10^{12} \cdot 1.3 \cdot 10^{18}}{5.1 \cdot 10^6} = 2.5 \cdot 10^{23}$$

REFERENCES

1. Kaon Physics with a High Intensity Proton Driver, Belyaev, et. al., hep-ph/0107046.
2. G. D'Ambrosio, G. Ecker, G. Isidori, and J. Portoles, JHEP 08 (1998) 004.
3. J.R. Batley, et. al., Phys. Lett. **B576**, 43 (2003).
4. I derived this angle of 57° from equation 1. G. Isidori, in private communication, suggested me to use a similar value.
5. Rick Coleman's program, based on the Malensek parameterisation, to estimate proton flux needed for kaon production at various energies and targetting angles.